Folium of Descartes

This is a famous curve with a long history (see e.g. `http:\\www-history.mcs.st-andrews.ac.uk/Curves/Curves.html`). The curve is the solution set of the equation

$$x^3 + y^3 = 3axy.$$

One can see that the solutions for different a differ only by scaling, namely divide the equation by a^3 and replace $x/a,\ y/a$ by x, y.

The two most frequently given parametrizations are:

$$x(t) = \frac{3t}{1+t^3},\ y(t) = \frac{3t^2}{1+t^3},$$
$$r(\varphi) = \frac{\sin 2\varphi}{\sin^3\varphi + \cos^3\varphi},\quad -\pi/4 < \varphi < 3\pi/4.$$

The first parametrization has the disadvantage that at $t = -1$ the denominator vanishes, the curve jumps “from minus infinity to plus infinity”, while the important double point at $0 \in \mathbb{R}^2$ is left out (or given by $t = \infty$). This can be remedied by the transformation $u = 1/(1+t),\ t = -1 + 1/u$, which changes the parametrization to

$$x(u) = \frac{u^2 - u^3}{1 - 3u + 3u^2},\ y(u) = \frac{u - 2u^2 + u^3}{1 - 3u + 3u^2},$$
$$-\infty < u < \infty.$$

H.K.